

Probability Axioms and Set Theory Paradoxes

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Abstract: In this paper, we show that Zermelo–Fraenkel set theory with Choice (ZFC) conflicts with basic intuitions about randomness. Our background assumptions are the Zermelo–Fraenkel axioms without Choice (ZF) together with a fragment of Kolmogorov’s probability theory. Using these minimal assumptions, we prove that a weak form of Choice contradicts two common sense assumptions about probability—both based on simple notions of symmetry and independence.

Keywords: set theory; probability; axiom of choice

1. A Puzzle

We begin with a paradox involving the Axiom of Choice (AC) and an infinite set of fair coins. An early version of this result first appeared as a problem in the American Mathematical Monthly [1]; a version closer to ours can be found in [2].

Let $I = 2^{\omega}$ denote the set of all binary-valued functions on $\omega = \{0, 1, 2, \dots\}$. Let $\psi : I \rightarrow I$ be a randomly constructed function. By this, we mean that for each $r \in I$, $\pi \in \omega$, $\psi(r)(\pi)$ is determined by a fair coin toss. (If you prefer, you may also interpret the elements of I as the binary expansion of a real number $0 \leq r \leq 1$. We note that each dyadic rational, $0 < r < 1$, can occur in two ways; e.g., $\frac{1}{2} = 0.10 = 0.0\bar{1}$.) We now ask: “If $P \in I$ is chosen at random (i.e., $P(\pi)$ is determined by a fair coin toss for each $\pi \in \omega$), is it possible to guess the value of $\psi(P)$ given the values $\psi(r)$ for all $r \neq P$?” The intuitively obvious answer is “no”. Since each value of ψ was chosen independently, the restriction of ψ to $I \setminus \{P\}$ should carry no information about $\psi(P)$. Hence, if we are limited to information about $\psi \upharpoonright (I \setminus \{P\})$, the odds of guessing $\psi(P)$ correctly should be 0, no matter what strategy we employ.

But are they? Consider the following argument, which can be fully formalized within Zermelo–Fraenkel set theory with Choice (ZFC). Define an equivalence relation on I^I by setting $f \sim g$ if and only if $f(r) = g(r)$ for all but finitely many $r \in I$. By AC, there exists an $S \subseteq I^I$ that intersects each \sim -class in one point. For each $g \in I^I$, let g^* be the unique function in $S \cap [g]$, and let Δ_g denote the finite set $\{r \in I : g(r) \neq g^*(r)\}$. Note that g^* is uniquely determined by the restriction of g to any cofinite subset of I . Hence, ψ^* is determined by $\psi \upharpoonright (I \setminus \{P\})$, so we can employ the strategy of guessing that $\psi(P) = \psi^*(P)$. This strategy fails if and only if $P \in \Delta_{\psi^*}$. Since Δ_{ψ^*} is a finite set depending only on ψ (not on P), any randomly chosen $P \in I$ is almost certain not to lie in Δ_{ψ^*} . Thus, using only information about $\psi \upharpoonright (I \setminus \{P\})$, our strategy is almost certain to guess the correct value of $\psi(P)$. This paradox invites us to reexamine the assumptions underlying ZFC.

2. Introduction

2.1. Axioms and Mathematical Intuition

Over the last 100 years, set theory (ZFC) has become widely accepted as a foundation for mathematics. If mathematics is to be a search for objective truth, then the correctness of these foundational axioms is essential. Axiomatic systems, such as ZFC, turn our mathematical intuitions into precise statements. Hence, their validity ultimately depends on whether these intuitions are correct. Further, mathematics will always have meaningful



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Paradoxes In Probability Theory And Mathematical Statistics

Emilie Sanchez



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